Music, Philosophy, and Natural Science in the Middle Ages

Miyazaki International College
André Goddu

Introduction and General Background

In the European Middle Ages, philosophy was studied and developed in a framework that was largely circumscribed by Greco-Roman cosmological traditions and Judaeo-Christian-Islamic theological commitments. Medieval cosmology and theology also informed and shaped developments in music and musical theory. The exact sciences continued traditions of mathematical modeling and observation, but they also did so in ways that were shaped by the world-view of Greco-Roman antiquity within the limits allowed by theology and orthodox practice.

What may appear initially as constraints, however, also provided structure, background, in brief, much that is common between music, natural philosophy, and science. In music the dominant tradition was Pythagorean and Platonic with its greater emphasis on rational ideals rather than on empirical ones, and the force of that tradition lasted several centuries. Indeed, the force of rational ideals continues to influence western efforts to understand the universe. As conceived in predominantly rational terms, music in ancient and medieval western contexts was a branch of mathematics, the branch that examines and studies numerical ratios and proportions. Like astrology, music was regarded as a science that reveals the rational order in the cosmos. As a science, music was thought to provide the fundamental principles of cosmic harmony. In the ratios of simple whole numbers and their multiples lay the relationships between all sorts of natural phenomena, especially astronomical and medical. Consequently, the possibility of discovering the musical laws that govern the whole of creation, and especially the celestial universe, intoxicated and motivated mathematicians, music theorists, and astronomers for centuries.

The musical consonances admitted in Pythagorean theory arise out of the *tetraktys* (1, 2, 3, and 4), restricting the consonant intervals to the simple ratios, 2:1, 3:2, 4:3, or to compounds of these ratios. The Pythagorean conception of harmony, however, also required placing a third term, or mean, between two terms of a given ratio, generating a numerical system that preserved the arithmetic proportion (4:*3*:2 or 12:*9*:6) and the harmonic proportion (6:*4*:3 or 12:*8*:6). From the combination of these terms (12:*9*:*8*:6), all of the interlocking ratios and the geometric proportion (4:*2*:1) can be generated, limited by the octave (the extremes 12:6) and by the whole tone (the means 9:8). Later Pythagorean thinkers saw in these proportions and their multiples the mathematical basis for all sorts of natural phenomena.[1]

As taken up by Boethius in *De institutione musica*, Pythagorean theory was supplemented with a summary of Ptolemy's *Harmonica*.[2] Although the Pythagorean system accepts only three ratios as consonant: the octave (2:1), the fourth (4:3), and the fifth (3:2), Ptolemy objected to the strictly arithmetical grounds for rejecting obviously consonant sounds as dissonant.[3] Ptolemy appealed to acoustics to justify a less rigid and more graduated distinction between consonance and dissonance.[4] Ptolemy's objections had little effect, yet his viewpoint prevailed eventually among musicians and composers. While musical theory became more mathematical and less musical, actual musical practice introduced a tension into the mathematical theory of music that was every bit as troublesome as that between the Aristotelian theory of the heavens and Ptolemaic astronomy and between the Aristotelian account of visual perception and perspectivist accounts of vision.

Allow me to summarize briefly the incompatibility between Aristotelian theory and, first, Ptolemaic astronomy and, second, the perspectivist account of light and vision developed by Ibn al-Haytham (Alhazen) and his followers, and how medieval thinkers reconciled these differences.[5] To put the principal differences in a nutshell, we refer to differences that were scandalous for Aristotelians——in astronomy between Aristotle's literally exact geocentrism and Ptolemy's approximate geocentrism, and——in theories of light and vision between the instantaneous actualization of the medium in illumination and the fact that propagation and transmission of *species in medio* across a medium implied some passage of time. In astronomy, the most elegant compromises involved embedding the Ptolemaic eccentric and epicycle models in homocentric spheres or orbs. Such systems of nested spheres came to dominate the European understanding of Ptolemy until the seventeenth century. In theories of vision, the most elegant compromises involved spiritual or spatio-temporal interpretations of *species* that rendered them incorporeal and hence compatible with instantaneous theories of light transmission. Even Kepler seems to have retained the notion of *species* in this form.[6)]

Despite the apparent success of these compromises, they merely delayed the seemingly inevitable confrontation with Aristotelianism that they were destined to provoke. The fact is that the Aristotelian theories both in astronomy and on light and vision were empirically and mathematically deficient. As long as there were natural philosophers concerned with adequate descriptions of the phenomena, the efforts of Aristotelians to save their theories could not eliminate altogether the tension between theory and data. Again, Kepler serves to illustrate the point both in astronomy and in theories of light and vision, where his rejection of Aristotle is clear and explicit.[7)]

Although the compromises that were achieved were far from perfect in resolving the tensions between Aristotle and Ptolemy and between Aristotle and perspectivist theory, the traditions kept the problems alive. They contributed to the dissatisfaction and frustration that motivated reform and, eventually, to the proposal of new solutions in the sixteenth and seventeenth centuries.

The example of music is similar although a break between theory and practice occurred already in the Middle Ages, or so some of us have been arguing. In the fourteenth century, according to this view, a series of events occurred that have been obscured by scholarly focus elsewhere, namely on astronomy and problems of motion. Scholars looking for continuity with early modern science focused on the cosmology and natural philosophy of the fourteenth century.[8)] Later, we were told that the developments of the fourteenth century were not as influential or as anticipatory of modern science as we were once taught.[9)] The contexts of fourteenth-century discussions were pedagogical and logical exercises meant to challenge and develop the minds of students, not motivate them to discover new truths about the physical universe.[10]

Distracted by the exaggerations of Pierre Duhem and then by the critiques of Anneliese Maier, John Murdoch, and Edith Sylla, most scholars focused on philosophical matters or pedagogical and social factors and neglected the one case where practical developments challenged standard theory and led to new theoretical developments.[11] If the subject were not so technical and still puzzling from a modern perspective, its neglect might be considered a scandal. The fact is that we are still struggling to understand these developments, and we are still trying to find explanations that are both satisfactory and persuasive.

Some fundamental modern studies of medieval music begin with an investigation of music as a specifically *medieval* science. Max Haas, Dorit Tanay, Christopher Page, Alfred Crosby, and others have shown that the standard issue in the Middle Ages was classification, the relation of music to other sciences. Music was classified as one of the *scientiae mediae*, partly mathematical and partly physical. Music theorists, philosophers, and others who discussed music brought to their discussions ideas and methods from other sciences——logic, ethics, and perception; medicine, physiology, and emotions; astronomy, arithmetic, and geometry; theology, liturgy, and biblical studies. Although adequate attention to the physical dimension was suppressed for several centuries, discussions of music had cosmological implications that amplified even the most reductive of analyses and that implied broader foundations.[12]

In short, as with every other principal subject in medieval thought, the part mirrored the whole, that is, the discussion of a part entailed discussion of relations to other parts and to the whole. Music, then, was a microcosm of the whole of knowledge, and through examination of that microcosm we gain access to authors' conceptions of science both in the broad philosophical sense and in the narrower sense of an exact science.

Still, implications merely announce possibilities and potentialities. For potentialities to become actual, new conditions must be introduced, and it seems that those new conditions appeared in the thirteenth century. The transformation was perhaps predictable from Ptolemy's criticisms of Pythagorean harmonic theory, but the first medieval evidence of a change from the strict and rigid adherence to the mathematical, arithmetical, and proportional dogmatism of Pythagorean theory appeared in the thirteenth century under the influence of Aristotelian natural philosophy. The assimilation of Aristotelian natural philosophy led to the transformation of music into a *scientia media* and stimulated the changes that occurred in the composition and performance of music in the late thirteenth and fourteenth centuries. The transformation generated a problem that demanded a theoretical response, and indeed there was one.[13] Today, the sorts of new approaches and the recognition of the problems mentioned above are provoking deeper and wider examinations of music treatises and of the relation between music and other disciplines. For my part, I have focused my attention on developments in logic, and on the use of logic in natural philosophy, musical theory, and the exact sciences.

There are especially two themes that I wish to develop here today: 1) medieval developments in the logic of consequences and their relation to the harmonic conceptions that led Copernicus to the discovery of the order of the planets and to Kepler's discovery of the harmonic law, and 2) the idea that the concept of musical consonance can be applied to the knowability of creation: the order of creation makes knowledge possible.[14] The claim has epistemological and metaphysical significance, but as a claim it is grounded in the logic of dialectical topics. I will attempt to clarify that logical ground today.

Harmony in Astronomy

For over thirty years the historiography of the Copernican Revolution was dominated by Thomas Kuhn's holistic paradigm theory.[15] According to the theory, competing paradigms are incommensurable, which means that there can be no logical transition from the one to the other. Once we select the new paradigm, there is no going back to the older one. Compromises are also excluded for there can be no process of comparison that would lead to a compromise.

Although Kuhn had studied the Copernican Revolution closely, his own theory did not fit the Copernican Revolution.[16] The misfit was obvious, in fact. One of the major competitors for some seven decades was a geo-heliocentric compromise that combined the best features of geocentric and heliocentric theories. It was obvious that the two theories could be compared. If comparable, then why could there not be a critique that could persuade one of the superiority of one over the other? If a critique was possible, then why should we not be able to evaluate the logic of the critique and the logic of the arguments supporting the competing theories?

In his arguments against geocentrism and in support of heliocentrism, Copernicus placed great emphasis on such criteria as harmony, symmetry, and commensurability. These criteria have been interpreted primarily as aesthetic in nature, and thus as appealing to the emotions in a rhetorical fashion. The links between traditional mathematical and musical conceptions of the universe and Copernicus's arguments, then, tended to be dismissed as not strictly rational, and so his conclusions have often been judged to be nonlogical and even illogical.

Harmony and Part-Whole Relations in Logic

The logic of consequences is concerned with the relationship between the antecedent and consequent of a conditional proposition. There are many rules that help us to construct valid consequences, and a standard source and teaching tool in the Middle Ages for learning these rules was a handbook by Peter of Spain called the *Summulae logicales*. Peter also wrote a text entitled *Syncategoremata*, a second important source for the medieval logic of consequences. In addition to Peter, two texts by William of Sherwood were also important sources for the medieval doctrine of consequences.[17)]

These authors along with terminist logicians developed theories that may not be formally rigorous by mathematical standards but that are suited to explaining the consequences that are used in natural language and in different disciplines.[18)] One of the contexts and problems that contributed to the development of such broader theories of consequence was the logical status of the so-called paradoxes of strict implication:

(1) from the impossible anything follows, and
(2) the necessary follows from anything.[19)]

Regarded in a purely formal way, the paradoxes are merely the result of a formal calculus, that is, they follow from certain laws of entailment that we need in order to construct a chain of proofs.[20] For the logician concerned with natural contexts or the requirements of a specific discipline, however, the paradoxes are counterintuitive. William of Sherwood, Peter of Spain, and medieval logicians complained that because a conclusion is a proposition proved by an argument, and because an argument provides grounds and warrants for believing the conclusion, then it cannot be true to say that from the impossible anything whatever follows or to say that the necessary follows from anything whatever. The inferences leading to conclusions must be confirmed by certain intrinsic, extrinsic, or intermediate topics.[21] These logicians did not conclude, however, that nothing at all follows from the impossible or even the false, because there are contexts where the antecedent of a consequence is a counterfactual and where the truth of the consequence is some sort of hypothetical or causal proposition. From a purely formal truth-functional analysis of counterfactuals we would conclude that they must all be true. Only if we add some semantic criterion, where we must assert some relation between antecedent and consequent, can we judge some counterfactuals to be true and others false.[22]

There are many dialectical topics, rules, or relations that can mediate between a premise and a conclusion and between the antecedent and consequent of a conditional proposition, but let me draw your attention to two topics in particular: the intrinsic topics "from an integral whole" and "from an integral part." Peter of Spain says of these topics:

> An Integral Whole is what is composed of parts having quantity and a part of it is called integral. The topic from Integral Whole is a relation between a thing itself and its part. It is always constructive, for example: "*there is a house, therefore there is a wall.*" The topic here? From an Integral Whole. The Maxim: given an Integral Whole, any part of it is given as well.
>
> The Topic from Integral Part is a relation between a thing itself and its whole. It is always destructive. For example, "*there*

is no wall; therefore there is no house." The Topic here? From an Integral Part. The Maxim: given rejection of an Integral Part, its Whole is rejected as well.[23)]

I emphasize these topics and maxims above all because they play an enormous role in Copernicus's arguments on behalf of heliocentrism and they express the central role that harmony plays in Copernicus's view of the cosmos.

The Use of Logic and Dialectical Topics in Astronomy

The planets must be in a particular and definite order, but how can we discover that fact? Ptolemaic theory provides no unique principle for the ordering of the planets. When Copernicus placed the sun in the center, he noticed that the planets, including Earth, can be ordered according to their respective sidereal periods, and that this arrangement accounts for the observation of the bounded elongations of Mercury and Venus (because the orbits of these planets are *interior* to Earth's orbit) and for the observation of the retrograde motions of all of the planets and why the superior planets undergo retrograde motion only when in opposition. The part-whole logic of Copernicus's argument takes the following form: The whole is the system of planets centered on the sun and ordered according to sidereal periods (the period of each planet's realignment with a given star), and the parts are the orbits of each of the planets around the sun. Even so, Copernicus's conception of harmony here was a very general one, namely, that the parts of the system are commensurable, that is, the planets are ordered in a definite way according to one principle, sidereal periods. Still, Copernicus did not discover any mathematical law or ratios that reduced the entire system to a harmonic law. That discovery comes later, as we shall see; nevertheless, the logical warrant for Copernicus's inference that there is a whole system in which the parts are ordered according to a common measure is the dialectical topic "from an integral whole."

There is yet a second important clue for the logical connection between musical harmony and astronomy, namely, the logical ground for asserting the knowability of the order and motions of the planets. Just as in medieval logic and the doctrine of dialectical topics where logical or real relations supply the ground linking antecedent and consequent and linking the steps in an argument, so also ideas of system contributed to the belief that the order of creation makes knowledge of creation possible.[24] Because God created both nature and human reason, some authors argued, then it must be possible for the human mind to understand nature.[25] There must be a harmonious relationship between the human mind and nature, such that when we discover a reason or explanation for known but otherwise inexplicable facts, we must be accessing or approaching the organizing principle behind creation itself.

Copernicus criticized Ptolemaic astronomers above all for their failure to produce a harmonious system. His arguments rely especially on part-whole relationships and on the criterion of relevance between the antecedent and consequent of hypothetical propositions in astronomy. He complained bitterly about the scandalous result that after a thousand years, mathematical astronomers still could not decide in what unique order God had arranged the planets. The full argument seems to be that the observed motions could belong to the whole or to the part or to both. Ptolemy argues from the observations made on Earth that the motions belong to the whole. Copernicus implies that Ptolemy's conclusion is the result of a fallacy, the fallacy of using the topic from integral part *constructively*.[26]

Rheticus too appealed to the divinely created order of nature and to Copernicus as the astronomer who had discovered the correct tuning principles of the cosmic instrument.[27] Rheticus implicitly compared the systems of Copernicus's opponents to an instrument made by God that had no tuning principles whatsoever and that produced a cosmic cacophony. As a matter of fact, Rheticus was mistaken about Copernicus's achievement, but Kepler was persuaded that heliocentrism provided the right clues and direction for discovering empirically the correct tuning principles of the cosmic instrument.[28] At first, the arguments by Copernicus and Rheticus struck me as rhetorical and persuasive rather than dialectical and probable, but I have since come to see them as grounded on the assumption of the harmonious relationship between the mind and the universe, that is to say, on the very notion of a *proportio* linking God, nature, and the human mind.

Copernicus appealed to God as the best and most systematic Artisan of all as the reason why we should be able to understand the movements of the world machine with greater certainty.[29] Evidently picking up on that theme, Rheticus appealed to God's skill as a craftsman in constructing a system that reduces an almost infinite number of appearances to the one motion of the earth.[30] They implicitly use the topic from efficient cause and its maxim, "that is good whose efficient cause is good as well."[31] Copernicus and Rheticus implied that there is harmony between the mind and nature and that the mind, therefore, possesses the capacity to know nature. Indeed, God created the world for our sake, says Copernicus.[32] These suggest topics from dignity, purpose, final cause, but above all they rest on the assumption of proportionality, of a universe created according to a regulative principle, and that principle is none other than the analogy between the order and motions of the planets under the control of the Sun and the proportionality of the parts of a painting or the ratios of a well-tuned instrument.

The inference is, at first glance, theological in character, and its warrant and backing rely on a tradition of allusions to divine, artistic, and musical ordering principles. On second glance, the inference relies on the belief that the mind has a natural capacity and affinity for explanations such that when it discovers them, it finds them pleasing and beautiful and so is moved to assent to their truth. Indeed, Copernicus's appeal to the order of nature and the human mind's capacity to see it was an explanation of why Copernicus was persuaded that he had discovered the truth. The key, however, was not the beauty of the theory itself. There are many beautiful but probably false theories. The key was the agreement between a beautiful theory and minutely precise data. In a rationally constructed universe, such agreement, so Copernicus, Rheticus, and Kepler believed, could not be accidental. The effectiveness and persuasiveness of a part-whole analysis rests on the primacy of the whole, whether it be a style of portraiture, a musical tone-system, or a planetary system. An analysis that reveals the ordering principle, capturing the harmony and commensurability of the parts in relation to the whole must be right, or so they thought.

As I mentioned earlier, it was left to Kepler to discover the harmonic law, and what a remarkable discovery it was, in part because Kepler himself gave it no special emphasis.[33] Copernicus discovered that the orbits of the planets constitute a system or whole that is regulated by the sun. Copernicus's "harmonic" conception is that there is a common principle (sidereal periods) ordering the planets linearly from the shortest to the longest sidereal periods. The harmonic law discovered by Kepler is that the square of the orbital period of a planet divided by the cube of a planet's mean distance from the sun is a constant. The whole system, then, is bound by that law, regulating the orbital period and mean distance of each planet. Each individual orbit is linked harmonically to the whole. The part-whole relationship serves as a warrant for Kepler's conclusion that the Copernican system is ordered and regulated by one harmonic law.

The Traditions in Music, Logic, Natural Philosophy, and Astronomy

Reformers like Copernicus and Kepler did not waste a great deal of time in separating themselves from their medieval predecessors. They used their works, recognized both their usefulness and their shortcomings, and proceeded to correct them and advance beyond them. It is, in fact, the very same attitude that they adopted to contemporaries whose works they also used and rejected.

It has been my purpose in this paper to portray the tensions in music theory and practice as another example of the tensions that needled astronomers and natural philosophers for centuries. I began this essay with a comparison. There were three tensions in medieval natural philosophy: between Aristotle and Ptolemaic astronomy, between Aristotle and perspectivist theories of light and vision, and between Pythagorean harmonic theories and actual musical practice. The resolution of these tensions, I have argued, rested on the following assumption, criteria, or beliefs: 1) agreement between theory and data; 2) that what is incidental in the old theory is explained by the new theory; 3) that the agreement between theory and data and the fact that the theory explained the data could not be accidental; and 4) the basic assumption about the consonance and harmony of the universe in relation to the human mind, backed by the belief in a good and wise creator who assures the knowability of his creation. The arguments that support the new and successful conclusion depended on a tradition of dialectical argumentation. The dialectical topics used as inference-warrants and backed by the assumption of consonance and harmony are what link medieval musical theory, natural philosophy, and science in the Middle Ages to early modern astronomy.

The exact science in the story has always been a matter of constructing a closer agreement between theory and data. What I have tried to elucidate are the arguments whereby music theorists, natural philosophers, and astronomers tried to persuade their contemporaries that they had achieved at least partial solutions. A study of the arguments reveals their use of logic, dialectical topics, and rhetoric. I have focused especially on the use of dialectical topics, and here I think it would be helpful to summarize those results.

The warrants that were applied most often (for example, in the arguments on whether the Earth or the Sun is central and immobile) are nobility or dignity, containing and contained, whole-part relationships, simplicity, order, symmetry, relativity, and, finally, the appropriateness of using the Bible to settle philosophical or scientific arguments.

To us today many arguments are of exclusively academic interest and may even appear silly, but in studying such arguments we are examining the deep-seated beliefs and assumptions of a community of scholars. Above all, we are witnesses to the very sea changes that we call revolutionary, the questioning of standard beliefs and the proposal of new ones, and the re-interpretation of the warrants used to link the new assumptions with the known data.

There are several traditions in logic that we could cite, but one of the standard handbooks of the Middle Ages was Peter of Spain's *Tractatus*, especially the treatise on topics, a discussion of the arguments that produce belief regarding a matter that is in doubt. A topic supports or confirms an argument. Topics are divided into maxims and differentiae, and the latter are divided into intrinsic, extrinsic, and intermediate. In other words, topics or inference warrants are distinguished into kinds according to whether the relation grounding the inference is intrinsic (e.g., the genus and species of a definition), extrinsic (e.g., opposites, contraries, authority), or intermediate (e.g., the division of an expression into its connotations).

The use of topics came to play an important role in the evaluation of conditional propositions and in supplying the missing premisses of an enthymeme. In the medieval analysis of conditional propositions there were always philosophers who insisted on some sort of connection between the antecedent and consequent of a conditional proposition. By the late fifteenth century, some authors were defending this view on the basis of a supposed relationship between the antecedent and consequent. For example, John of Glogovia, probably one of Copernicus's teachers in Cracow, mentions the following topics: essential superior to inferior, whole to essential part, integral whole to part, cause and effect, cause of following, and correlative. In addition to the standard intrinsic grounds, John of Glogovia enumerates inclusion and containment. What is signified by the consequent must be included in, contained by, or relevant to what is signified by the antecedent. Conversely, a good consequence admits nothing extraneous or irrelevant.

Copernicus's insistence on the connection between antecedent and consequent of a hypothetical proposition is based on the structure, coherence, and commensurability of the whole, and on relations such as whole and part, contained and containing, relevance, and the following of observations from hypotheses necessarily without the addition of anything extraneous or irrelevant.[34)]

The scientific developments in astronomy and musical acoustics produced theories regulated ever more precisely by ever more empirically accurate data. The transition to the more precise regulation of theories was accomplished in part by the construction and discussion of arguments that persuaded contemporaries that new theories provided partial solutions. The arguments were supported by the dialectical topics and inference-warrants developed during the Middle Ages, and so ends my case for the contribution of medieval logic to late medieval music, natural philosophy, and science and, not incidentally, to the Scientific Revolution of the seventeenth century.

NOTES

1) I have recently summarized these points with references elsewhere: A. Goddu, "Harmony, Part-Whole Relationships, and the Logic of Consequences, "*Musik-und die Geschichte der Philosophie und Naturwissenschaften*, ed. F. Hentschel, forthcoming (Berlin: W. de Gruyter, 1998). Regarded from a modern mathematical point of view, the whole number ratios of Pythagorean harmonic theory start out from the division of the octave of a string into the ratio 2:1. The ratios of the intervals of any major diatonic scale as produced by the Pythagorean monochord, multiplied together, equal 2. These were the mathematical proportions that were thought to make music possible. If we take the intervals of the pythagorean monochord for the C-major diatonic scale: D:C=9:8; E:D=9:8; F:E=256:243; G:F=9:8; A:G=9:8; B:A=9:8; C':B=256:243, and then divide the product of the numerators by the product of the denominators, the result is 2: (9x9x256x9x9x9x256) ÷ (8x8x243x8x8x8x243) = 2.

2) The fundamental study here remains Boethius, *De institutione musica*, tr. as *Fundamentals of Music* by C. Bower (New Haven and London: Yale University, 1989).

3) Cf. A. Goddu, "Music as Art and Science in the Fourteenth Century," *Scientia und ars im Hoch-und Spätmittelalter*, *Miscellanea mediaevalia* 22, 2 (1994) p. 1027.

4) See Bower (note 2), pp. xx-xxix, for confirmation of this point.

5) Some of the ideas presented here can be found with documentation in the following two papers: A. Goddu, "The Logic of Copernicus's Arguments and his Education in Logic at Cracow,"*Early Science and Medicine* 1 (1996) 28-68, esp. 28-31; and A. Goddu, "William of Ockham's 'Empiricism' and Constructive Empiricism," *Die Gegenwart Ockhams*, ed. W. Vossenkuhl and R. Schönberger (Munich: VCH, 1990), pp. 208-231, esp. 215-223. The point with respect to light and vision was originally made by D. Lindberg, "The Science of Optics," *Science in the Middle Ages*, ed. D. Lindberg (Chicago: University of Chicago, 1978), pp. 338-368, esp. 354.

6) To confirm these points, the reader may consult the following: Goddu, "Logic" (note 5) pp. 28-31; Nicolaus Oresme, *Expositio et Quaestiones in Aristotelis De anima*, ed. B. Patar with introduction by C. Gagnon (Louvain and Paris: Peeters, 1995), Book II, qq. 15-18, pp. 229-281; O. Nakamura, "Nicole Oresme's Epistemology——A Synthesis of Aristotle's *De anima* and *Perspectiva*," Kyoto University Society of Medieval Philosophy, 1 June 1997; and D. Lindberg, "The Genesis of Kepler's Theory of Light: Light Metaphysics from Plotinus to Kepler," *Osiris*, 2d series, 2 (1986) 5-42, esp. 33-42.

7) See Lindberg (note 6).

8) Of course, the classic studies here are those of Pierre Duhem: *Études sur Léonard de Vinci*, 3 series, (Paris: Hermann, 1906-1913); *Le système*

du monde, 10 volumes, (Paris: Hermann, 1954-1959).

9) The correspondingly classic critiques are those of Anneliese Maier, *Studien zur Naturphilosophie der Spätscholastik*, 5 volumes (Rome: Storia e Letteratura, 1949-1968); and *Ausgehendes Mittelnalters*, 3 volumes (Rome: Storia e Letteratura, 1964-1977).

10) See especially the study by John Murdoch, "From Social into Intellectual Factors: An Aspect of the Unitary Character of Late Medieval Learning," *The Cultural Context of Medieval Learing*, ed. J. Murdoch and E. Sylla, Boston Studies in the Philosophy of Science, 26, Synthese Library, 76 (Dordrecht and Boston: Reidel, 1975), pp. 271-339. Cf. also J. Murdoch, "Philosophy and the Enterprise of Science in the Later Middle Ages," *The Interaction between Science and Philosophy*, ed. Y. Elkana (Atlantic Highlands, N. J., 1974), pp. 51-74.

11) In fairness to and defense of Murdoch, we must mention a seminal article, "Music and Natural Philosophy: Hitherto Unnoticed *Questiones* by Blasius of Parma?," *Manuscripta* 20 (1976) 119-136. For further references, see A. Goddu, "Connotative Concepts and Mathematics in Ockham's Natural Philosophy," *Vivarium* 31 (1993) 106-139. We may also mention other studies that have tried to revive questions about the relation between medieval natural philosophy and early modern science: A. de Libera, "La développement de nouveaux instruments conceptuels et leur utilisation dans la philosophie de la nature au XIVe siècle," *Knowledge and the Sciences in Medieval Philosophy*, Proceedings of the Eighth International Congress of Medieval Philosophy, Helsinki 24-29 August 1987, *Acta Philosophica Fennica* 48 (1990) 158-197; and E. Grant, who though critical of Duhem retained some of Duhem's opinions, but now he has finally addressed these questions in a constructive fashion based on his own extensive familiarlity with the sources and contexts. See Grant's recent study, *The Foundations of Modern Science in the Middle Ages* (Cambridge: University Press, 1996).

12) Compare M. Haas, "Studien zur mittelalterlichen Musiklehre I: Eine Übersicht über die Musiklehre im Kontext der Philosophie des 13. und frühen 14. Jahrhunderts," *Aktuelle Fragen der musikbezogenen Mittelalterforschung, Forum Musicologicum, Basler Beiträge zur Musikgeschichte*, III (Winterthur: 1982); D. Tanay, "Music in the Age of Ockham," Ph. D. dissertation, University of California at Los Angeles, 1989; C. Page, *Discarding Images* (Oxford: University Press, 1993); and A. Crosby, *The Measure of Reality* (Cambridge: University Press, 1996).

13) Compare W. Hirschmann, "Die *Commendacio omnium scientiarum et specialiter Musice* im Musiktraktat der beiden Heilsbronner Mönche (1295) und ihre Beziehung zur philosophischen Einleitungliteratur," *Musik*, ed. F. Hentschel (note 1); E. Hirtler," 'numerus relatus'-'numerus sonorus'-'sonus numeratus'. Die 'musica' im Übergang von der 'scientia mathematica' zur

'scientia media'," ibid.; C. Panti, "Robert Grosseteste's Theory of Sound," ibid.; and M. Walter, "Über den musikalischen Begriff 'proportio'," ibid.

14) In his article, "Von Denkmustern, Konnotationen und Implikationen," *Archiv für mittelalterliche Philosophie und Kultur*, Heft III, ed. T. Boiadjiev et al. (Sofia, 1996), pp. 104–118, esp. 118, F. Hentschel expressed it thus: "Als erkennbar erweist sich die Schöpfung aufgrund ihrer Ordnung."

15) T. Kuhn, *The Structure of Scientific Revolutions* (Chicago, 1962).

16) T. Kuhn, *The Copernican Revolution* (Cambridge, Massachusetts, 1957).

17) Petrus Hispanus, *Syncategoreumata*, ed. L. M. de Rijk and tr. J. Spruyt (Leiden 1992); id., *Tractatus* (*Summule Logicales*), ed. L. M. de Rijk (Assen, 1972); William of Sherwood, *Introduction to Logic*, tr. N. Kretzmann (Minneapolis, 1966); id., *Treatise on Syncategorematic Words*, tr. N. Kretzmann (Minneapolis, 1968); K. Jacobi, "Drei Theorien über die Funktion aussagenverknüpfender Zeichen: Die Diskussion des Junktors 'si' bei Wilhelm von Shyreswood," *Sprache und Erkenntnis im Mittelalter*, *Miscellanea Mediaevalia* 13/1 1981) 385–402; A. Maierù, *University Training in Medieval Europe*, tr. D. N. Pryds (Leiden, 1994), pp. 93–116; and E. Morscher, "Der Begriff 'consequentia' in der mittelalterlichen Logik," *Archiv für Begriffsgeschichte* 15 (1971) 133–139.

18) S. Toulmin, *The Uses of Argument* (Cambridge, 1958), pp. 146–210.

19) I. Boh, "John of Glogovia's Rejection of Paradoxical Entailment Rules," *Die Philosophie im 14. und 15. Jahrhundert*, ed. O. Pluta (Amsterdam, 1988), pp. 373–383, esp. 373. Cf. A. N. Prior, "Logic, Modal," *The Encyclopedia of Philosophy*, ed. P. Edwards, Vol. 5 (New York, 1967), pp. 5–12.

20) Prior, *Logic* (note 19) pp. 6–7; Jacobi, *Theorien* (note 17) pp. 386–390.

21) Jacobi, *Theorien* pp. 394–396; Petrus Hispanus, *Syncategoreumata* (note 17) pp. 230–237.

22) Jacobi, *Theorien*, p. 396.

23) Petrus Hispanus, *Tractatus* (note 17) p. 64: "Totum integrale est quod est compositum ex partibus habentibus quantitatem et pars eius dicitur integralis. Locus a toto integrali est habitudo ipsius ad suam partem. Et est semper constructivus. Ut '*domus est; ergo paries est.*' Unde locus? A toto integrali. Maxima: posito toto integrali, ponitur quelibet eius pars. Locus a parte integrali est habitudo ipsius ad suum totum. Et est semper destructivus. Ut '*paries non est; ergo domus non est.*' Unde locus? A parte integrali. Maxima: destructa parte integrali, destruitur et suum totum." The translation cited is by F. P. Dinneen, *Language in Dispute: The Summulae Logicales* (Amsterdam, 1990), pp. 56–57.

24) Hentschel (note 14) p. 118: "Als erkennbar erweist sich die Schöpfung aufgrund ihrer Ordnung." I would render this statement literally as "Creation shows itself to be knowable as a consequence of its order."

25) Nicolaus Copernicus, *De Revolutionibus libri sex*, ed. R. Gansiniec, *Opera*

omnia, 2 vols., (Warsaw, 1973-78), Vol. 2, p. 4, lines 33-37: "Hanc igitur incertitudinem mathematicarum traditionum de colligendis motibus sphaerarum orbis cum diu mecum reuoluerem, coepit me taedere, quod nulla certior ratio motuum machinae mundi, qui propter nos ab optimo et regularissimo omnium opifice conditus esset, philosophis constaret, qui alioqui rerum minutissima respectu eius orbis tam exquisite scrutarentur." Georg Joachim Rheticus, *Narratio prima*, tr. E. Rosen, *Three Copernican Treatises*, 2d ed. (New York, 1959), p. 137.

26) Copernicus, *De Revolutionibus* (note 25) Vol. 2, p. 4, 33-37: "Hanc igitur incertitudinem mathematicarum traditionum de colligendis motibus sphaerarum orbis cum diu mecum reuoluerem, coepit me taedere, quod nulla certior ratio motuum machinae mundi,..." Goddu, "Logic" (note 5) pp. 43-61. For Copernicus's criticism: *De Revolutionibus*, Vol. 2, p. 11, 20-24: "Quam ob causam ante omnia puto necessarium, ut diligenter animaduertamus, quae sit ad caelum terrae habitudo, ne dum excelsissima scrutari volumus, quae nobis proxima sint ignoremus, ac eodem errore quae telluris sint attribuamus caelestibus." I have explained the context of the argument, though not the argument itself, more fully in "Logic" (note 5) pp. 54-55.

27) Rheticus, *Treatises* (note 25) pp. 138-139.

28) See J. V. Field, *Kepler's Geometrical Cosmology* (Chicago and London, 1988), pp. 145-179.

29) Copernicus, *De Revolutionibus*, Vol. 2, p. 4, 33-37: "... ab optimo et regularissimo omnium opifice conditus esset, ..."

30) Rheticus, *Treatises*, p. 137.

31) Petrus Hispanus, *Tractatus* (note 17) p. 67: "... cuius causa efficiens bona est, ipsum quoque bonum est." Translation by Dinneen (note 23) p. 59.

32) Copernicus, *De Revolutionibus*, Vol. 2, p. 4, 33-37: "... qui propter nos ab optimo et regularissimo omnium opifice conditus esset, ..."

33) See Field (note 28) pp. 143-144 and 162. Kepler himself seems to have realized how surprising the third law was, yet he treats it almost casually probably because the third law is "irrelevant," as Field puts it, "to the main astronomical concern of *Harmonices Mundi* Book V...", although it does play "an important part in the cosmological work of Book V," Chapter 9.

34) See "Logic," (note 5), pp. 49-61, for examples of integral whole; omitting something essential, admitting the irrelevant, i. e. relevance as a warrant; genus and species; essential properties; universal whole or genus; from efficient cause; authority; rejection of scriptural objections as incompetent; property, dignity and containing and contained in the argument relating form or shape to motion; proper motions indicate that the motions of the parts are independent of the whole and therefore cannot be attributed to the whole; the nobility and dignity of immobility and, therefore, it is more appropriate to attribute immobility to the sphere of the stars; the enclosing

and containing should remain immobile; the reduction of the order of the planets to one principle; the explanation of the observations as following necessarily, not incidentally, from the assumptions; parsimony; final cause; and, finally, the greater probability of the new assumptions.